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VARIATIONAL BOUND ANALYSIS OF A DISCONTINUITY IN NONRADIATIVE DIELECTRIC WAVEGUIDE.

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Abstract. This paper describes the application of the variational bound method to Nonradiative Dielectric waveguide, for the analysis of scattering by a dielectric obstacle, in this case an airfilled discontinuity in the dielectric centre strip. Closed form equations are obtained that can be used directly in the design of networks using reactive components, such as filters. Measured data agrees well with the theoretical calculations.

I INTRODUCTION.

The application of specific properties of discontinuities in waveguides forms the basis of a variety of microwave devices. In the Nonradiative Dielectric waveguide only one such analysis has been reported, by Yoneyama et al [1] , where a step discontinuity was described and applied in the design of a filter. Expressions for describing the network are not given.

In this paper, the variational bound (VB) method described by Aronson et al [2] is used to analize the scattering from a rectangular hole through the dielectric centre conductor of the NRD guide. A variational bound is obtained on η , the phase shift which measures the amount by which the asymptotic solution for the fields in the guide in the presence of the discontinuity is displaced relative to the guide without a discontinuity.

A major advantage of the procedure is that a closed form expression for the equivalent network is obtained. The design of networks making use of reactive elements is consequently greatly simplified, because it eliminates empirical determination of element values.

II ANALYSIS.

In order to make the analysis applicable, it is necessary to modify the waveguide cross section by allowing the waveguide modes as well as the surface mode to exist. Consequently, the waveguide is boxed by introducing a pair of metal walls along the open sides of the NRD. However, these are made to be analytically far enough away so that in practice this artifice does not affect the theoretical results. Fig. 1 shows an isometric view of the guide, and also defines the dimensions. The analysis closely follows that outlined in [2] . In the absence of the obstacle, the odd and even electric fields are given by,

$$\mathbf{E}_{Te} = \mathbf{e}_e(x,y) \cos k_z z \qquad (1)$$

$$\mathbf{E}_{To} = \mathbf{e}_o(x,y) \sin k_z z \qquad (2)$$

In the presence of an obstacle the asymptotic form ($z \to \infty$) of the fields are given by

$$\mathbf{E}_e = C_e \mathbf{e}_e(x,y) \; [-\sin k_z z + \cot\eta_e \cos k_z z] \quad (3)$$

$$\mathbf{E}_o = C_o \mathbf{e}_o(x,y) \; [\cos k_z z + \cot\eta_o \sin k_z z] \quad (4)$$

where η_e and η_o are the even and odd mode phase shifts associated with the discontinuity, and $\mathbf{e}(x,y)$ is the vector form function of the dominant TM_{y11} mode. This will be normalized such that

$$\int \mathbf{e}_s(x,y).\mathbf{e}_s(x,y) \, dxdy = 1, \qquad (5)$$

where the area of integration is defined by the walls of the waveguide, and $\mathbf{e}_s$ denotes the transverse part of $\mathbf{e}$. The description of the TM_{y11} mode is obtained from [3].

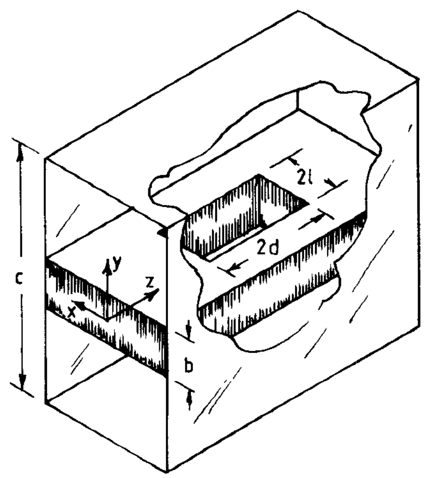

Fig 1. Isometric view of the rectangular hole in the waveguide centre dielectric strip.

To obtain the two phase constants, we solve the differential equation,

$$\frac{d^2 f}{dz^2} + f\,[\,k_z^2 - \int_{-\ell}^{\ell} \int_{b/2}^{b/2} (\varepsilon_r - 1) k_o^2 \, \mathbf{e.e} \, dxdy\,] = 0 \qquad (6)$$

The static differential equation (6) reduces to

$$d^2 f/dz \; - K^2 f = 0. \qquad (7)$$

Because the obstacle is symmetric, we can write,

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$$f = f_e + f_o, \text{ and} \tag{8}$$

$$f_e = D_e \cosh Kz, \quad f_o = D_o \sinh Kz. \tag{9}$$

If equations (9), (3) and (4) and their derivatives are equated at the discontinuity boundary, then the even and odd mode phase shifts are given by,

$$\cot \eta_e = \frac{-\sin k_z d + k_z/K \coth Kd \cos k_z d}{-k_z/K \sin k_z d \coth Kd - \cos k_z d} \tag{10}$$

$$\cot \eta_o = \frac{\cos k_z d + \sin k_z d + \sin k_z d \; k_z/K \tanh kd}{k_z/K \cos k_z d \tanh Kd - \sin k_z d}$$

where the wave number in the z-direction is k_z. The transverse wave numbers, and are solutions of the trancendental equation,

$$\beta \tan(\beta b/2) = \varepsilon_r \alpha \tag{12}$$

subject to the constraint

$$\alpha^2 + \beta^2 = k_o^2(\varepsilon_r - 1) \tag{13}$$

while the guide wave number is calculated from,

$$k_z^2 = k^2 - (\pi/a)^2 - \beta^2. \tag{14}$$

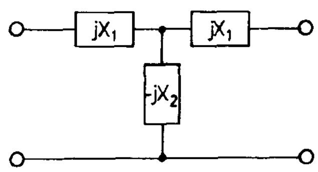

Fig. 2. Equivalent circuit of waveguide discontinuity.

The parameter K is calculated in closed form from,

$$K^2 = -k_z^2 + \frac{(\varepsilon_r - 1)k_0^2}{A} \cdot \{(k_z\beta)^2/\varepsilon_r \left[\ell + \frac{\sin(2\pi\ell/a)}{2\pi/a}\right] \cdot \left[b/2 - \frac{\sin(\beta b)}{2\beta}\right]$$
$$- \left[\frac{(k^2 - \beta^2)}{\varepsilon_r}\right]^2 \cdot \left[\ell + \frac{\sin(2\pi\ell/a)}{2\pi/a}\right]\left[b/2 + \frac{\sin(\beta b)}{2\beta}\right] - \left[\frac{\pi}{a} \cdot \frac{\beta}{\varepsilon_r}\right]^2$$
$$\left[\ell - \frac{\sin(2\pi\ell/a)}{2\pi/a}\right] \cdot \left[b/2 - \frac{\sin(\beta b)}{2\beta}\right]\}, \tag{15}$$

The normalizing constant is given by,

$$A = -\left(\frac{\pi}{a} \cdot \frac{\beta}{\varepsilon_r}\right)^2 \cdot \left(\frac{a}{2}\right) \cdot \left[b/2 - \frac{\sin(\beta b)}{2\beta}\right] - a\left[\frac{\pi}{a}\alpha \frac{\cos(\rho b/2)}{\cosh(\rho(c/2-b/2))}\right]^2$$
$$\cdot \left[-\frac{b}{2} + \sinh(\alpha(c/2-b/2)) . \frac{\cosh(\alpha(c/2-b/2))}{2}\right] - \left[\frac{k^2 - \beta^2}{\varepsilon_r}\right]^2 \cdot \left(\frac{a}{2}\right)$$
$$\cdot \left[\frac{b}{2} + \frac{\sin(\beta b)}{2\beta}\right] - a\left[k_o^2 + \alpha^2\right] \cdot \left[\frac{\cos(\beta b/2)}{\cosh(\alpha(c/2-b/2))}\right]^2 \cdot$$
$$\left[\frac{b}{2} + \frac{1}{2\alpha} \sinh(\alpha(c/2-b/2)) . \cosh(\alpha(c/2-b/2))\right] \tag{16}$$

It is now possible to calculate the equivalent admittance network for holes of any given dimension. The element values for the equivalent T-network shown in Fig. 2 are calculated as, [2]

$$X_1 = \tan \eta_o \tag{17}$$

$$X_2 = \tfrac{1}{2}(\cot \eta_o + \tan \eta_e). \tag{18}$$

III EXPERIMENTAL RESULTS.

The reflection coefficient and phase angle of the equivalent T-section of two rectangular holes, of dimensions 10 x 20 mm and 10 x 10 mm respectively, were measured, and the data is plotted in Fig. 3 (a) and (b) together with the theoretically calculated values using the equations developed above. The agreement between the various sets of values are good.

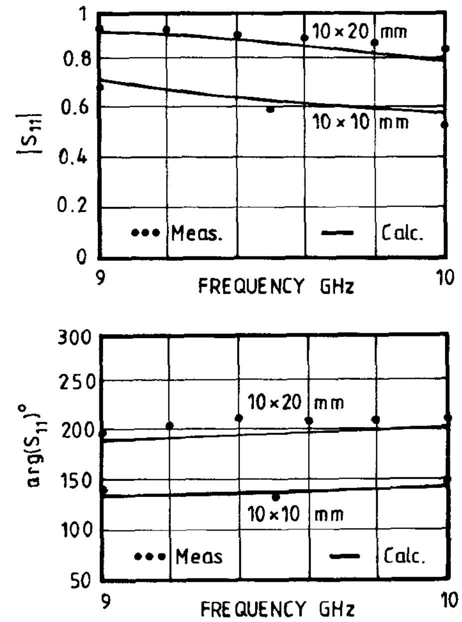

Fig. 3. Measured (dotted lines) and calculated values (solid lines) for the reflection coefficient and phase of the equivalent circuit.

IV CONCLUSION.

A theoretical analysis that makes it possible to calculate the equivalent circuit for a rectangular discontinuity in NRD waveguide, has been presented and verified experimentally. A major advantage of the theory is that the equations are in closed form.

References.

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